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Macrodynamics and the Demise of the Swedish School

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179-WORD ABSTRACT

The founding father of the Swedish school was Wicksell, who in 1898 used a macroeconomic dynamic disequilibrium method to set out a cumulative process of inflation at frozen physical output. Using Wicksell's method, Ohlin in 1934 added physical output as an additional variable and verbally described a multiplier-accelerator interaction. In 1937 Lundberg wrote the difference equations of such an interaction and solved them recursively. Prompted by Hansen, Samuelson in 1939 recovered the primitive underlying such difference equations. Finally, using continuously distributed lags, Phillips in 1954 wrote the differential equations of a multiplier-accelerator interaction and recovered their primitive.

Underlying Swedish period analysis was the conviction that lags were important. Differential equations may look less "Swedish" than difference equations but handled lags at least as well--as Phillips showed. So the length of the period doesn't matter and may even vanish! If so, the lasting contribution of the Swedish school cannot have been its use of periods of finite length but rather must have been the rise the Swedes gave to the use of more efficient forms of macrodynamics.

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1. A Fifty-Year Perspective

The Swedish[1] school is identified with economic dynamics, micro-
economic as well as macroeconomic. Its founding fathers were Wicksell
(1851-1926) and Cassel (1866-1945). Wicksell's macrodynamic accomplish-
ment was his [1898 (1936)] short-run cumulative process of inflation
at frozen physical output. Cassel's microdynamic accomplishment was
his [1918 (1932: 32-41 and 137-155)] dynamization of a Walrasian
general economic equilibrium into a model of a "uniformly progressing
state" growing in its physical quantities at frozen prices. His macro-
dynamic accomplishment was his [1918 (1932: 61-62)] aggregation of
such a model. His microdynamics inspired von Neumann; his macro-
dynamics anticipated Harrod.

As practical people, economists are in the habit of first doing
their work and only later, if at all, reflect on how they did it.
Swedes are no exception. One might distinguish between an early
Swedish school doing the work and typified by Myrdal (1927), Lindahl
(1930), Ohlin (1934), and Lundberg (1937) and a later one reflecting
on methodology and typified by Svennilson (1938) and Lindahl (1939,
part I). A recent lucid monograph by Petersson (1987) makes such a

distinction, takes stock after 50 years have passed, and closes with the question: has the Swedish school died?

If so, why? In its attempt to generalize, the later Swedish school probably overgeneralized, losing both operational significance and touch with reality. Long before econometrics, Sweden had a proud tradition for measurement--after all, her population census of 1749 was by far the oldest in the world. Wicksell [1919 (1934: 255)] had paid tribute to Cassel's massive use of data: "it is in my opinion incomparably the best part of his work. Professor Cassel's great gifts for concrete description based on facts and figures here show to advantage." Despite its Casselian heritage the Swedish school never established any liaison with econometrics, the up and coming link with reality. Worse still, reality itself began to move away from the fundamental disequilibria of the mass unemployment of the thirties and the suppressed inflation of the forties to which the Swedish method had been so closely tailored. The new reality of fine-tuned smooth growth in the fifties and sixties demanded no disequilibrium method. The seventies and eighties may have brought disequilibria back but nowadays, as Petersson observes in closing, period analysis is out of fashion. Indeed it is, but why?

Let us follow our own line of reasoning and begin with a simple question: why do schools die, anyway?

2. Inherent Logical Impasse

A school may reach the end of its line because of an inherent logical impasse, as did the labor theory of value: first, relaxing its assumption that in every good factors combine in the same proportion would bring down the theory as a house of cards. Second, at best the theory could only explain relative price. Even under constant unit cost, demand would always affect relative quantity produced, i.e., affect allocation. On allocation the theory was condemned to silence as long as it refused to make room for preferences. The Swedish school certainly did not die of such an inherent logical impasse. But a school may reach the end of its line because a superior alternative becomes available. Superior in what sense?

3. Alternative Has Superior Richness

If Keynes won and the Swedes lost, was it because he had a richer model that could do something they could not do? True, Keynes froze his price and let physical output be his equilibrating variable. True, a founding father of the Swedish school was Wicksell (1898) who had done the opposite, i.e., frozen his physical output and let price be his equilibrating variable. But Wicksell had given the Swedes not

only his particular application but also his method. That method was
fundamentally new in three respects: it was macroeconomic, it was
dynamic. and it was a disequilibrium method based upon adaptive expec-
tations whose disappointment constituted the motive force of the
system.

4. Multiplier-Accelerator Interaction: Words (Ohlin)

Using Wicksell's method and inspired by Lindahl's (1930) refine-
ment of it, Ohlin (1933), (1934) added physical output as an addi-
tional variable. Two years ahead of Keynes, Ohlin used three Keynesian
tools, i.e., the propensity to consume, liquidity preference, and the
multiplier, and one non-Keynesian tool, i.e., the accelerator. The four
tools would interact as follows in a feedback mechanism. Let consump-
tion demand be stimulated. As a result physical output would rise,
generating new income. The propensity to consume would link physical
consumption to the level of physical output and thus establish a con-
sumption feedback. The accelerator would link physical investment to
the growth of physical output and thus establish an investment feed-
back. As did the Wicksellian one, Ohlin's two feedbacks unfolded in a
cumulative process along a time axis as a succession of disequilibria:
expectations and plans were forever being revised in the light of new

experience. By contrast, Keynes used only the consumption feedback and telescoped it into an instant static equilibrium along an output axis.

Was Keynes's model richer, then? On the contrary: Ohlin had the richer model and went Keynes one better by using the accelerator, which gave him a feedback and cumulative process both missing in Keynes's macrostatics. Still Keynes won and the Swedes lost.

5. Alternative Has Superior Operational Significance

Richness isn't everything, operational significance also matters. And sometimes richness may stand in the way of operational significance. Keynes won on his operational significance: there was so much you could do with his model, and you could do it so sure-footedly! Ironically, precisely because Ohlin's model was richer, he was facing a multitude of possible sequences of his cumulative process.

Ohlin used nothing but words. His conscientious, accurate, cautious, and honest words certainly made no attempt to hide his multitude of possibilities. Worse, his words did not, and perhaps could not, sort out the possibilities and specify the exact circumstances under which each would materialize. As a result, his readers came away with the impression that anything could happen. And away

they went--to Keynes! To sum up, the instrument Ohlin had chosen to
communicate with his readers was too blunt.

6. Multiplier-Accelerator Interaction: Recursive Solution (Lundberg)

Would a keener instrument have served better? A big step forward
was Lundberg's (1937) period analysis. Lundberg wrote the difference
equations and solved them recursively for five cases.

First Lundberg set out a pure Keynesian multiplier: all invest-
ment was autonomous, and the system would eventually settle down in a
Keynesian stationary equilibrium--but Lundberg had traced the time
path to such an equilibrium.

Second, Lundberg set out the interaction between such a multiplier
and an accelerator based on a constant working-capital coefficient
called k and set equal to $1/2$. The example used was a capital stock of
inventory of "raw materials and goods in process."

Third, Lundberg set out the interaction between the multiplier
and an accelerator based on a constant fixed-capital coefficient called
μ and alternatively set equal to 15, 20, or 30. The example used was
a capital stock of residential housing. The larger the μ the more
powerful the expansion generated.

Fourth, to get away from such mechanical accelerators Lundberg
introduced the rate of interest and made his capital coefficient μ a
function of it. The rate of interest, in turn, was determined endo-
genously within the sequence by the supply of saving and the demand
for investment. As a result, the powerful expansion generated in the
third case had "been completely broken down when the influence of
variability in the rate of interest is taken into consideration"
(1937: 223).

Fifth, Lundberg returned to the interaction between a multiplier
and a mechanical accelerator based on a constant fixed-capital coef-
ficient still called μ and now set equal to 4. Here the example used
was a capital stock of machines, but machines of a very special kind.
The example was intended to show the effect of labor-saving "ration-
alization," hence new machines were assumed to need no additional
labor at all.

By writing his difference equations, estimating his parameters
numerically, solving his system recursively, and discussing the pro-
perties of his solutions, Lundberg achieved a high degree of opera-
tional significance. All that remained was to recover the primitive
that gave rise to such difference equations. That was done before the
thirties were out. What prompted it was the accelerator.

7. The Accelerator

For his accelerator Lundberg had credited Clark (1917), Frisch (1931), and Kalecki (1935, (1936) and could, of course, also have mentioned Cassel's [1918 (1932: 61-62)] use of it in an algebraically specified macroeconomic growth model anticipating the Harrod (1948) model by 30 years.

Containing a derivative with respect to time, the accelerator is inherently dynamic and will dynamize any system including it, Keynesian or non-Keynesian. Not for long, then, could Keynes's disciples remain confined to his statics. One of them was Alvin Hansen who in 1927 had surveyed business-cycle theories and later (1951: ix) always emphasized "the vast importance of the Continental development of the theory of investment demand and the role of investment in income formation--the work of Wicksell, Tugan-Baranowsky, Spiethoff, Schumpeter, and Cassel--a development largely overlooked by English-speaking economists." According to Samuelson (1976: 29-30), a "Minnesota visit of Frisch in 1931 was important for Hansen's quick integration of the acceleration principle into the Keynesian system." By the time Hansen came to Harvard in 1937 he liked to see physical investment as the change in desired physical capital stock and, in

turn, see desired physical capital stock in direct proportion to physical output of consumers' goods. Like Lundberg at the same time, Hansen tried to work out the arithmetic of an interaction between the multiplier and the accelerator. Bewildered by the resulting multitude of possibilities, Hansen turned to his brightest student for help.

8. Multiplier-Accelerator Interaction: Primitive Recovered (Samuelson)

That student was Samuelson (1939) who wrote the following system:

Variables

$C \equiv$ physical consumption

$I \equiv$ physical investment

$X \equiv$ physical output

Parameters

$b \equiv$ capital coefficient, the accelerator

$c \equiv$ propensity to consume

$G \equiv$ physical government purchase of goods and services

The system had only three equations. First, the lagged consumption function

$$C(t) = cX(t - 1), \tag{1}$$

where c is the propensity to consume. Second, the lagged investment function

$$I(t) = b[C(t) - C(t - 1)], \tag{2}$$

where b is the accelerator. Third, the goods-market equilibrium condition

$$X(t) = C(t) + I(t) + G \tag{3}$$

For our purposes we may ignore Samuelson's government purchase G, insert (1) and (2) into (3) thus collapsed, and write his reduced system as the linear homogeneous second-order difference equation

$$X(t) + AX(t - 1) + BX(t - 2) = 0 \tag{4}$$

where the coefficients were

$$A \equiv - (1 + b)c$$

$$B \equiv bc$$

Try a solution of the form $X(t) = x^t$ whose growth factor is x and whose growth rate is $\log_e x$ where x is a constant to be found. Insert that form into (4), divide by x^{t-2}, and find the "characteristic" or "auxiliary" quadratic

$$x^2 + Ax + B = 0 \tag{5}$$

Our tentative solution $X(t) = x^t$ is then a solution if and only if x is a root of (5). Since (5) is a quadratic there are two such roots, hence two solutions X(t). A weighted sum of them will be the primitive. A particular solution of (4) can be obtained from the primitive by determining the weights in accordance with the initial conditions of the system. If the roots of (5) were complex, physical output would display oscillations. If they were not, physical output would converge to a stationary state or be growing smoothly.

Here Keynes's brightest disciple had recovered the primitive underlying an Ohlin-Lundberg-type cumulative process and thus given the latter more operational significance than it ever had in Sweden.

At 24, had Samuelson heard of the Swedes? He had and "mentioned in passing that the formal structure of our problem is identical with the model sequences of Lundberg..."

Samuelson opened up a new era of macrodynamics. Perhaps the era would not have opened up so quickly had it not been for Baumol's (1951) magnificent text. At 29, had Baumol heard of the Swedes? He had. His text grew out of lectures at the London School of Economics in 1947-1949 and paid tribute to Ralph Turvey who knew Swedish, con- tributed a chapter and an appendix, and (1951: ix) "made his influence felt throughout the volume."

Lundberg and Samuelson used difference equations, i.e., equations using periods of finite length. Their finite length make them a mathematical transcription of verbal Swedish period analysis--a transcription never used by the otherwise mathematical father of the cumulative process, Wicksell himself.[2] Could what Lundberg and Samuelson did also have been done by differential equations? Phillips (1954) almost did it--we recommend Allen's (1956: 72-74) more straightforward summary.

Differential equations require lags to be continuously distributed, and Phillips used a continuously distributed investment lag corres- ponding to Samuelson's rigid lag (2). But Phillips used a lagless consumption function. Instead he had a continuously distributed

supply lag. For complete comparability with Samuelson we must rewrite
Phillips as follows.

9. Multiplier-Accelerator Interaction: Differential Equations
    (Phillips Rewritten)

To rewrite Phillips (1954) Samuelson's notation will do except for
two new parameters:

$\alpha \equiv$ speed of response of consumption to output

$\beta \equiv$ speed of response of investment to change in output

First, let there be a continuously distributed lag in the response
of consumption to output. Specifically let the response dC/dt of con-
sumption be in proportion to the gap between desired and current con-
sumption. Desired consumption, in turn, is the propensity to consume
c _times_ current output. Consequently:

$$\frac{dC}{dt} = \alpha(cX - C) \tag{6}$$

Next, let there be a continuously distributed lag in the response of investment to the change in output. Specifically, let the response dI/dt of investment be in proportion to the gap between desired and current investment. Desired investment, in turn, is the accelerator b _times_ current change in output. Consequently:

$$\frac{dI}{dt} = \beta(b\,\frac{dX}{dt} - I) \tag{7}$$

As in Samuelson, ignore the supply lag and let goods-market equil-ibrium be

$$X = C + I \tag{8}$$

Now let us solve our system. Differentiate (8) with respect to time and insert (6) and (7) into the derivative. Then write (8) as C = X - I, insert that, rearrange, and arrive at I expressed in X alone:

$$I = \frac{1 - b\beta}{\alpha - \beta}\,\frac{dX}{dt} + \frac{\alpha(1 - c)}{\alpha - \beta}\,X \tag{9}$$

Notice that (9) would be meaningless if the speeds of response α and β were equal. Assuming they are not, differentiate (9) with respect to time and arrive at dI/dt expressed in X alone:

$$\frac{dI}{dt} = \frac{1 - b\beta}{\alpha - \beta} \frac{d^2X}{dt^2} + \frac{\alpha(1 - c)}{\alpha - \beta} \frac{dX}{dt} \tag{10}$$

Inserting (9) and (10) into the right-hand and left-hand sides, respectively, of (7) will finally give us the linear homogeneous second-order differential equation

$$\frac{d^2X}{dt^2} + A \frac{dX}{dt} + BX = 0 \tag{11}$$

where the coefficients are

$$A \equiv \frac{\alpha(1 - c) + (1 - \alpha b)\beta}{1 - b\beta}$$

$$B \equiv \frac{\alpha\beta(1 - c)}{1 - b\beta}$$

Try a solution of the form $X = e^{xt}$ whose growth factor is e^x and whose growth rate is x where x is a constant to be found. Insert that form into (11), divide through by e^{xt}, and find the very same form of the characteristic or auxiliary quadratic as in the case of difference equations:

$$x^2 + Ax + B = 0 \tag{12}$$

Our tentative solution $X = e^{xt}$ is then a solution if and only if x is a root of (12). Since (12) is a quadratic there are two such roots, hence two solutions X. A weighted sum of them will be the primitive. A particular solution of (11) can be obtained from the primitive by determining the weights in accordance with the initial conditions of the system.

10. Difference Versus Differential Equations

As it happened, most of the new-era macrodynamics used differential rather than difference equations--with Bent Hansen (1951) differential equations even sneaked into Sweden!

Their periods of finite length, we said, made difference equations a mathematical transcription of verbal Swedish period analysis. Does their replacement by differential equations mean the disappearance of the last trace of Swedish period analysis?

Underlying Swedish period analysis was always the conviction that lags were important. Samuelson's interaction had two lags, i.e., (1) and (2) in it , which gave him the second-order difference equation (4). Our Phillips-like interaction had the same two lags, only in continuously-distributed form, i.e., (6) and (7), which gave us the second-order differential equation (11). Both procedures could handle lags. If anything, differential equations handled their lags better than the rigid difference equations: with their freedom of choice of the response coefficients α and β continuously distributed lags were more flexible and more realistic.

Summing up, although they look less "Swedish" than difference equations, differential equations handle lags at least as well and have a characteristic or auxiliary equation of the very same form--as

Baumol (1951) and Lancaster (1968) have taught us they will. Once the roots of that equation have been found a primitive can be recovered governing the time path of our variable X.

11. The Earlier and the Later Swedish School

Finding that time path was what the earlier Swedish school was trying to do. Lindahl (1930) tried. Ohlin (1934) tried. Using recursive solutions Lundberg succeeded.. What did the later Swedish school do?

Instead of following up the success of the earlier Swedish school by recovering the primitives invisibly governing the time paths of Lindahl's, Ohlin's, and Lundberg's variables--as Samuelson, Phillips, and others have done--the later Swedish school [Svennilson (1938) and Lindahl (1939, part I)] lost itself in methodology. Some of its questions were relevant, e.g., what was a plan? or how were expectations formed? Others were irrelevant, e.g., how long should the period be? If difference and differential equations can both handle lags then length of period doesn't matter and may even vanish. In retrospect the copious discussion of that length [also by myself, Brems (1944)] was redundant.

12. <u>Conclusion</u>

If length of period doesn't matter and may even vanish, the lasting contribution of the Swedish school cannot have been its use of a period of finite length. Swedish period analysis was one form of macrodynamics, soon crowded out by other, more rigorous, forms.

Rather, the lasting contribution of the Swedish school was its early insistence that macrodynamics was important, was its early attempt--however intuitive--to make it work, and was the rise it gave to the use of more rigorous forms. In those forms a Swedish heritage may be traced, as the present paper has tried to do.

FOOTNOTES

[1]Following American usage I refer to the "Swedish" school. Swedes themselves refer to the "Stockholm" school.

[2]_Geldzins und Güterpreise_ was Wicksell's only nonmathematical book but could easily have been written and solved in difference equations-- how easily is shown, e.g., by Brems (1986, ch. 8).

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